• Introduction

This article is the continuation of part I. It deals with larger boards. The “productive” patterns are : 4NAQ, 4AQ and 1AQ… excepted for N = 22 for which our best placement unexpectedly uses pattern 1AQ. In the following whenever pattern 1AQ (or variant) is used, we show only the top left corner of the N x N board.

THREE SOLUTIONS FOR N = 19 ; U(19) = 132

S1 1AQ

```
Q Q Q
Q Q Q
Q Q Q Q Q
Q Q Q Q Q
```

S2 4AQ

```
Q Q Q Q
Q Q Q Q
Q Q Q Q
Q Q Q Q
Q Q Q Q
```

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These three placements have a first diagonal symmetry.

N.B : In the following : for \( N \geq 21 \) only reduced size boards are shown : the relevant information is to be found in the corner(s).

**ELEVEN SOLUTIONS FOR \( N = 20 \) : \( U(20) = 145 \)**
TWO SOLUTIONS FOR \( N = 21 \) : \( U(21) = 170 \)

\[
\begin{array}{ccc}
\text{4AQ} & & \\
\text{4NAQ} & & \\
\end{array}
\]

A SOLUTION FOR \( N = 22 \) : \( U(22) = 186 \)

Pattern: 1NAQ; this pattern has provided so far best solutions only for \( N = 12 \) and \( N = 22 \).
If a 23 rd queen is added on the cross and the pattern is placed on the 23 x 23 board, it provides only \( U = 210 < 216 = U(23) \).

**A SOLUTION FOR N = 23 : U(23) = 216**

![Diagram](image-url)

First diagonal symmetry

This case was a hard nut to crack! We conjecture that there exists no solution with:

\[ 211 < U < 216. \]

**FOUR SOLUTIONS FOR N = 24 : U(24) = 240**

<table>
<thead>
<tr>
<th>S1</th>
<th>4AQ</th>
<th>S2</th>
<th>4AQ</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image-url" alt="Placement S1" /></td>
<td><img src="image-url" alt="Placement 4AQ" /></td>
<td><img src="image-url" alt="Placement S2" /></td>
<td><img src="image-url" alt="Placement 4AQ" /></td>
</tr>
</tbody>
</table>

Placements S1 has a vertical axial symmetry. S2 and S4 have a first diagonal symmetry.
THREE SOLUTIONS FOR $N = 25$ : $U(25) = 260$

These 3 placements have no symmetry.
TWO SOLUTIONS FOR $N = 26 : U(26) = 290$

TWO SOLUTIONS FOR $N = 27 : U(27) = 324$
TWO SOLUTIONS FOR N = 28 : $U(28) = 360$

SIXTEEN SOLUTIONS FOR N = 29 : $U(29) = 381$
On the 29 x 29 board above, 30 queens are placed leaving 420 unattacked squares. Removing any one of these queens provides a solution for $N = 29$ with $U(29) = 381$. Due to the symmetry of the above placement, one can obtain this way: $30/2 = 15$ different solutions. Adding the 1AQ pattern solution, we obtain altogether 16 solutions.

**THREE SOLUTIONS FOR $N = 30$ : $U(30) = 420$**

**4AQ**

Vertical axial symmetry

**4AQ**

Vertical axial symmetry

**1AQ**

First diagonal symmetry
THIRTY SIX SOLUTIONS FOR N = 31 : U(31) = 442

To the left above 32 queens are placed on the 31 x 31 board, leaving 442 = U(31) unattacked cases. By removing any one of these 32 queens, a solution is obtained. Since the above placement has no symmetry, 32 different solutions can be obtained. Altogether: 32 + 2 = 34 solutions. Also Johan Claes recently sent us the two basic solutions below, with pattern 4NAQ:
A SOLUTION FOR N = 32

U(32) = 485

Vertical axial symmetry

There exists many suboptimal solutions with U = 484.

A SOLUTION FOR N = 33

U(33) = 530

First diagonal symmetry
TWO SOLUTIONS FOR N = 34

U(34) = 554

Variants of 1AQ

SIX SOLUTIONS FOR N = 35 ; U(35) = 602
THREE SOLUTIONS FOR N = 36

U(36) = 650

4AQ

Horizontal axial symmetry

First diagonal symmetry

No symmetry

A SOLUTION FOR N = 37

U(37) = 702

First diagonal symmetry
TWO SOLUTIONS FOR N = 38

U(38) = 731

Variant 1AQ

TWO SOLUTIONS FOR N = 39

U(39) = 785

Variant of 1AQ

FOUR SOLUTIONS FOR N = 40

U(40) = 841

Horizontal axial symmetry
A SOLUTION FOR $N = 41$

$U(41) = 873$

A SOLUTION FOR $N = 42$

$U(42) = 932$
TWO SOLUTIONS FOR N = 43

U(43) = 993

TWO SOLUTIONS FOR N = 44

U(44) = 1056

A SOLUTION FOR N = 45

U(45) = 1091
TABLE of U(N) : 4 ≤ N ≤ 45

<table>
<thead>
<tr>
<th>N</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
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<tbody>
<tr>
<td>U(N)</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>11</td>
<td>18</td>
<td>22</td>
<td>30</td>
<td>36</td>
<td>47</td>
<td>56</td>
<td>72</td>
<td>82</td>
<td>97</td>
</tr>
<tr>
<td>Nb sol</td>
<td>25</td>
<td>1</td>
<td>3</td>
<td>38</td>
<td>7</td>
<td>7</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>1</td>
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<tbody>
<tr>
<td>U(N)</td>
<td>111</td>
<td>132</td>
<td>145</td>
<td>170</td>
<td>186</td>
<td>216</td>
<td>240</td>
<td>260</td>
<td>290</td>
<td>324</td>
<td>360</td>
<td>381</td>
<td>420</td>
<td>442</td>
</tr>
<tr>
<td>Nb sol</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>3</td>
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<td>16</td>
<td>3</td>
<td>34</td>
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</table>

<table>
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<th>34</th>
<th>35</th>
<th>36</th>
<th>37</th>
<th>38</th>
<th>39</th>
<th>40</th>
<th>41</th>
<th>42</th>
<th>43</th>
<th>44</th>
<th>45</th>
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</thead>
<tbody>
<tr>
<td>U(N)</td>
<td>485</td>
<td>530</td>
<td>554</td>
<td>603</td>
<td>650</td>
<td>702</td>
<td>731</td>
<td>785</td>
<td>841</td>
<td>873</td>
<td>932</td>
<td>993</td>
<td>1056</td>
<td>1091</td>
</tr>
<tr>
<td>Nb sol</td>
<td>1</td>
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<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
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<td>1</td>
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</tbody>
</table>

CONCLUSION

The problem is still open : prove wether the placements for N ≥17 are optimal or not …

Also for each optimal value of U(N), find the number of different “basic” placements of N queens leaving U(N) unattacked squares.

Also for N tending to infinity, which is (are) the best types (patterns)of placements …

Thanks to Johan Claes for providing us many new placements of his own.